Obfuscation from LWE? proofs, attacks, candidates

Hoeteck Wee

CNRS & ENS

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

$$C$$

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

$$\begin{array}{c} C \\ \big\downarrow \\ \mathcal{O}(C) \end{array}$$

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

$$C \equiv C'$$

$$\forall x : C(x) = C'(x)$$

$$\mathcal{O}(C)$$

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

$$C \qquad \equiv \qquad C'$$

$$\forall x : C(x) = C'(x)$$

$$\mathcal{O}(C) \approx_c \mathcal{O}(C')$$

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

status. It's complicated

- candidates

  [GGHRSW13, GGH13, CLT13, BR13, BGKPS14, CLT15, ...]

- attacks

  [CHLRS15, CGHLMRST15, CLLT16, CLLT17, ADGM17, CGH17, ...]

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

status. It's complicated

CRYPTO complete

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

status. It's complicated

crypto COMPLETE

$\Rightarrow$ functional encryption

full domain hash

unrestricted fully homomorphic encryption

hardness of Nash equilibrium

obfuscation

[BGIRSVY01, H00, GR07, GGHRSW13]

from **LWE** *?*

candidates, proofs, and attacks

preliminaries

LWE assumption [Regev 05]

$$(\mathbf{A},\ \mathbf{sA}+\mathbf{e}) \approx_c \text{uniform}$$

LWE assumption [Regev 05]

$$(\mathbf{A},\ \mathbf{SA}+\mathbf{E}) \approx_c \text{uniform}$$

$$\boxed{\mathbf{S}}\ \boxed{\quad\mathbf{A}\quad} + \boxed{\quad\mathbf{E}\quad}$$

LWE assumption [Regev 05]

$$(\mathbf{A},\ (\mathbf{I}_2 \otimes \mathbf{S})\mathbf{A} + \mathbf{E}) \approx_c \text{uniform}$$

$$\boxed{\begin{matrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{matrix}}\ \boxed{\quad\mathbf{A}\quad} + \boxed{\quad\mathbf{E}\quad}$$

LWE assumption [Regev 05]

$$(\mathbf{A},\ (\mathbf{I}_2 \otimes \mathbf{S})\mathbf{A} + \mathbf{E}) \approx_c \text{uniform}$$

$$\boxed{\begin{matrix} \mathbf{S} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{matrix}}\ \boxed{\begin{matrix} \overline{\mathbf{A}} \\ \underline{\mathbf{A}} \end{matrix}} + \boxed{\mathbf{E}}$$

LWE assumption [Regev 05]

$$(\mathbf{A},\ (\mathbf{I}_2 \otimes \mathbf{S})\mathbf{A} + \mathbf{E}) \approx_c \text{uniform}$$

$$\boxed{\begin{matrix} \mathbf{S}\overline{\mathbf{A}} \\ \mathbf{S}\underline{\mathbf{A}} \end{matrix}} + \boxed{\mathbf{E}}$$

LWE assumption [Regev 05]

$$(\mathbf{A},\ (\mathbf{M} \otimes \mathbf{S})\mathbf{A} + \mathbf{E}) \approx_c \text{uniform}$$

$$\boxed{(\mathbf{M} \otimes \mathbf{S})\mathbf{A}} + \boxed{\mathbf{E}}$$

for any **permutation** matrix $\mathbf{M}$

LWE assumption [Regev 05]

$$(\mathbf{A},\ \underset{\sim\sim\sim\sim\sim\sim\sim}{(\mathbf{M}\otimes\mathbf{S})\mathbf{A}}) \approx_c \text{uniform}$$

$$\boxed{(\mathbf{M}\otimes\mathbf{S})\mathbf{A}} + \boxed{\quad\mathbf{E}\quad}$$

for any **permutation** matrix $\mathbf{M}$

branching programs

$$\begin{matrix} \mathbf{M}_{1,0} & \mathbf{M}_{2,0} & \cdots & \mathbf{M}_{\ell,0} \\ \mathbf{M}_{1,1} & \mathbf{M}_{2,1} & \cdots & \mathbf{M}_{\ell,1} \end{matrix}$$

$$\in \{0,1\}^{\text{poly} \times \text{poly}}$$

branching programs

$$\begin{array}{cccc} \boxed{\mathbf{M}_{1,0}} & \mathbf{M}_{2,0} & \cdots & \boxed{\mathbf{M}_{\ell,0}} \\ \mathbf{M}_{1,1} & \boxed{\mathbf{M}_{2,1}} & \cdots & \mathbf{M}_{\ell,1} \end{array}$$

evaluation. accept iff $\mathbf{M}_{\mathbf{x}} = \prod \mathbf{M}_{i,x_i} = \mathbf{0}$

branching programs

$$\begin{array}{cccc} \boxed{\mathbf{M}_{1,0}} & \mathbf{M}_{2,0} & \cdots & \boxed{\mathbf{M}_{\ell,0}} \\ \mathbf{M}_{1,1} & \boxed{\mathbf{M}_{2,1}} & \cdots & \mathbf{M}_{\ell,1} \end{array}$$

evaluation. accept iff $\mathbf{M}_\mathbf{x} = \prod \mathbf{M}_{i,x_i} = \mathbf{0}$

– read-many $\mathbf{M}_\mathbf{x} = \prod \mathbf{M}_{i,x_{i+1 \bmod n}}$, $|x| = n \ll \ell$

branching programs

$$\begin{matrix} \boxed{\mathbf{M}_{1,0}} & \mathbf{M}_{2,0} & \cdots & \boxed{\mathbf{M}_{\ell,0}} \\ \mathbf{M}_{1,1} & \boxed{\mathbf{M}_{2,1}} & \cdots & \mathbf{M}_{\ell,1} \end{matrix}$$

evaluation. accept iff $\mathbf{M}_\mathbf{x} = \prod \mathbf{M}_{i,x_i} = \mathbf{0}$

- read-many $\mathbf{M}_\mathbf{x} = \prod \mathbf{M}_{i,x_{i+1 \bmod n}}$, $|x| = n \ll \ell$
- captures both logspace and NC^1

branching programs

$$\begin{array}{ccccc} \boxed{\mathbf{u}} & \boxed{\mathbf{M}_{1,0}} & \mathbf{M}_{2,0} & \cdots & \boxed{\mathbf{M}_{\ell,0}} \\ & \mathbf{M}_{1,1} & \boxed{\mathbf{M}_{2,1}} & \cdots & \mathbf{M}_{\ell,1} \end{array}$$

evaluation. accept iff $\mathbf{u}\mathbf{M}_{\mathbf{x}} = \mathbf{u} \prod \mathbf{M}_{i,x_i} = \mathbf{0}$

- read-many $\mathbf{M}_{\mathbf{x}} = \prod \mathbf{M}_{i,x_{i+1 \bmod n}}$, $|x| = n \ll \ell$
- captures both logspace and NC^1

branching programs

$$
\begin{array}{cccc}
(1-a_1) & (1-a_2) & \cdots & (1-a_\ell) \\
(a_1) & (a_2) & \cdots & (a_\ell)
\end{array}
$$

evaluation. accept iff $\mathbf{M}_{\mathbf{x}} = \prod \mathbf{M}_{i,x_i} = \mathbf{0}$

example. (1×1 matrices)

branching programs

$$\begin{array}{cccc} (1-a_1) & (1-a_2) & \cdots & (1-a_\ell) \\ (a_1) & (a_2) & \cdots & (a_\ell) \end{array}$$

evaluation. accept iff $\mathbf{M}_{\mathbf{x}} = \prod \mathbf{M}_{i,x_i} = \mathbf{0}$

example. accept iff $\mathbf{x} \neq \mathbf{a}$ (1×1 matrices)

obfuscation

FIRST principles

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$\mathbf{M}_{1,0}$ $\mathbf{M}_{2,0}$

$\mathbf{M}_{1,1}$ $\mathbf{M}_{2,1}$

evaluation. $\mathbf{M}_{\mathbf{x}}$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0} \qquad \mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0}$$

$$\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1} \qquad \mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1}$$

evaluation. $\mathbf{M}_{\mathbf{x}}$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0}$ $\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0}$

$\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1}$ $\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1}$

evaluation. $\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}}$ $\boxed{(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}}$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}(\ \mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0} \qquad) \qquad\qquad \mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0}$$

$$\mathbf{A}_0^{-1}(\ \mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1} \qquad) \qquad\qquad \mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1}$$

evaluation. $\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}}$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$\mathbf{A}_0$ need a trapdoor to sample short pre-image of $\mathbf{A}_0$

$\mathbf{A}_0^{-1}(\ \mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0}\ )$ $\quad$ $\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0}$

$\mathbf{A}_0^{-1}(\ \mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1}\ )$ $\quad$ $\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1}$

evaluation. $\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}}$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0}) \quad)$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1}) \quad)$$

evaluation. $\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}}$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2)$$

evaluation. $(\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}})\mathbf{A}_\ell$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

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[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

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evaluation. $\underbrace{(\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}})\mathbf{A}_\ell}$ $\mathbf{M}_{i,b}, \mathbf{S}_{i,b}$ small **[ACPS09]**

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

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evaluation. $(\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}})\mathbf{A}_\ell \approx \mathbf{0}$

$$\iff \mathbf{M}_{\mathbf{x}} = \mathbf{0}$$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2)$$

evaluation. $(\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}})\mathbf{A}_\ell \approx \mathbf{0} \Rightarrow$ accept

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$(\mathbf{u} \otimes \mathbf{I})\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2)$$

evaluation. $(\mathbf{u}\mathbf{M}_\mathbf{x} \otimes \mathbf{S}_\mathbf{x})\mathbf{A}_\ell \approx \mathbf{0} \Rightarrow$ accept

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

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$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2)$$

candidate obfuscation for NC^1 !

[GGHRSW13, HHRS17, ...]

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$(\mathbf{u} \otimes \mathbf{I})\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}(\underset{\sim}{(\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\underset{\sim}{(\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2})$$

$$\mathbf{A}_0^{-1}(\underset{\sim}{(\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\underset{\sim}{(\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2})$$

Q. $\mathcal{O}(\mathbf{u}, \{\mathbf{M}_{i,b}\}) \overset{?}{\approx}_c \mathcal{O}(\mathbf{u}', \{\mathbf{M}'_{i,b}\})$

if $(\mathbf{u}, \{\mathbf{M}_{i,b}\}) \equiv (\mathbf{u}', \{\mathbf{M}'_{i,b}\})$

obfuscation via GGH15

[Gentry Gorbunov Halevi 15, Canetti Chen 17, ...]

$$(\mathbf{u} \otimes \mathbf{I})\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}(\utilde{(\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\utilde{(\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2})$$

$$\mathbf{A}_0^{-1}(\utilde{(\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\utilde{(\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2})$$

Q. $\mathcal{O}(\mathbf{u}, \{\mathbf{M}_{i,b}\}) \overset{?}{\approx}_c \mathcal{O}(\mathbf{u}', \{\mathbf{M}'_{i,b}\})$

if $\forall \mathbf{x} : \mathbf{u}\mathbf{M}_\mathbf{x} = 0 \iff \mathbf{u}'\mathbf{M}'_\mathbf{x} = 0$

$$\textsf{all}\ \left(\mathbf{u}, \{\mathbf{M}_{i,b}\}\right)$$

all reject

$\forall \mathbf{x} : \mathbf{u}\mathbf{M}_\mathbf{x} \neq \mathbf{0}$

some accept

all reject

$\forall \mathbf{x} : \mathbf{u}\mathbf{M}_{\mathbf{x}} \neq \mathbf{0}$

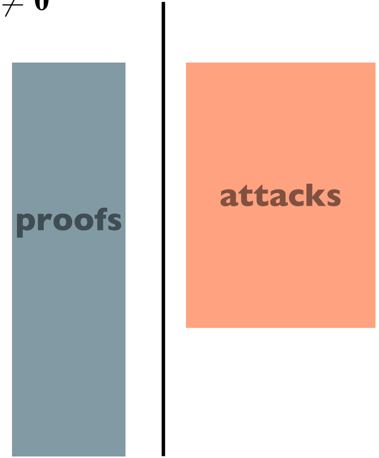
some accept

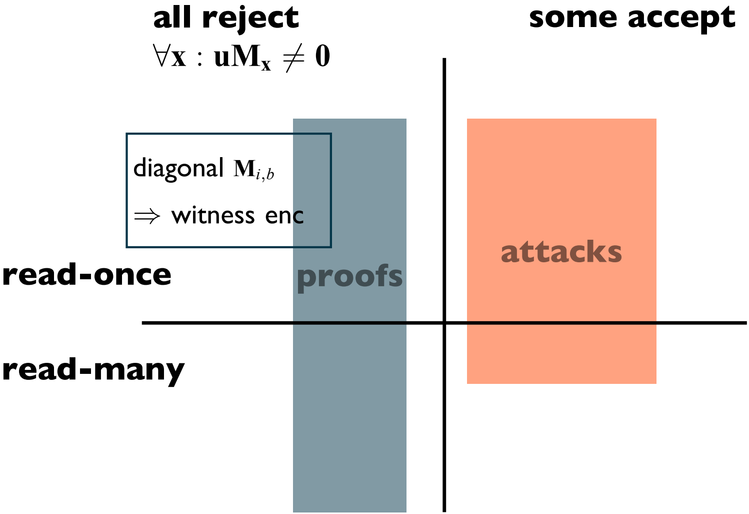
attacks

all reject

$\forall x : uM_x \neq 0$

some accept

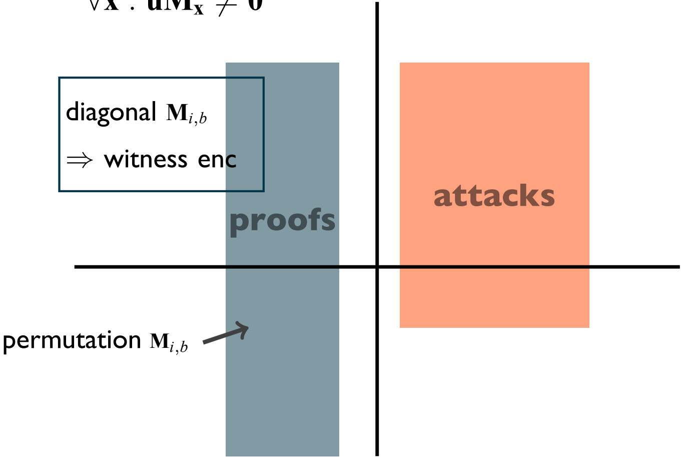



all reject
$\forall \mathbf{x} : \mathbf{u}\mathbf{M}_{\mathbf{x}} \neq \mathbf{0}$
some accept
diagonal $\mathbf{M}_{i,b}$
$\Rightarrow$ witness enc
read-once
proofs
attacks
read-many

all reject

$\forall \mathbf{x} : \mathbf{u}\mathbf{M}_{\mathbf{x}} \neq \mathbf{0}$

some accept



all reject

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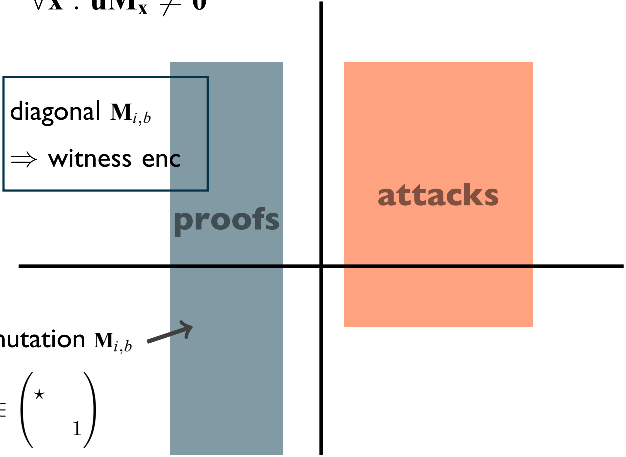
some accept



permutation $\mathbf{M}_{i,b}$

$$\mathbf{M}_{i,b} \in \begin{pmatrix} \star & \\ & 1 \end{pmatrix}$$

all reject

$\forall \mathbf{x} : \mathbf{u}\mathbf{M}_{\mathbf{x}} \neq \mathbf{0}$

some accept

diagonal $\mathbf{M}_{i,b}$

$\Rightarrow$ witness enc

proofs

attacks

permutation $\mathbf{M}_{i,b}$

$\mathbf{M}_{i,b} \in \begin{pmatrix} \star & \\ & 1 \end{pmatrix}$

candidate

NC^1 obfuscation

all reject

$\forall \mathbf{x} : \mathbf{u}\mathbf{M}_{\mathbf{x}} \neq \mathbf{0}$

some accept

1 **proofs**

permutation $\mathbf{M}_{i,b}$

$\mathbf{M}_{i,b} \in \begin{pmatrix} \star & \\ & 1 \end{pmatrix}$

2 **attacks**

candidate 3

NC^1 obfuscation

➊ proofs

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}(\underbrace{(\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\underbrace{(\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2})$$

$$\mathbf{A}_0^{-1}(\underbrace{(\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\underbrace{(\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2})$$

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

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$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

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lemma. $\approx_c$ random, for **permutation** matrices

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

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$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

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corollaries.

- private constrained PRFs **[Canetti Chen 17]**
- lockable obfuscation **[Goyal Koppula Waters, Wichs Zirdelis 17]**
- traitor tracing **[Goyal Koppula Waters 18, CVWWW 18]**

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$$\mathbf{A}_0$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2)$$

lemma. $\approx_c$ random, for **permutation** matrices

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$$

$$\mathbf{A}_0^{-1}(\underbrace{(\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\underbrace{(\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2})$$

$$\mathbf{A}_0^{-1}(\underbrace{(\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}(\underbrace{(\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2})$$

lemma. $\approx_c$ random, for **permutation** matrices

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$$

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lemma. $\approx_c$ random, for **permutation** matrices

proof. $\longleftarrow$ **[BVWW16]**

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,0} \otimes \mathbf{S}_{2,0})\mathbf{A}_2)$$

$$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1) \quad \mathbf{A}_1^{-1}((\mathbf{M}_{2,1} \otimes \mathbf{S}_{2,1})\mathbf{A}_2)$$

lemma. $\approx_c$ random, for **permutation** matrices

proof. $\longleftarrow$ **[BVWW16]**

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$

$\mathbf{A}_0^{-1}(\underbrace{(\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}$(uniform)

$\mathbf{A}_0^{-1}(\underbrace{(\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1}) \quad \mathbf{A}_1^{-1}$(uniform)

lemma. $\approx_c$ random, for **permutation** matrices

proof. $\longleftarrow$ [BVWW16]

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$$

$$\mathbf{A}_0^{-1}\big(\underbrace{(\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1}\big) \quad \mathbf{A}_1^{-1}(\text{uniform})$$

$$\mathbf{A}_0^{-1}\big(\underbrace{(\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1}\big) \quad \mathbf{A}_1^{-1}(\text{uniform})$$

lemma. $\approx_c$ random, for **permutation** matrices

proof. $\longleftarrow$ **[BVWW16]**

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$

$\mathbf{A}_0^{-1}((\mathbf{M}_{1,0} \otimes \mathbf{S}_{1,0})\mathbf{A}_1)$ uniform

$\mathbf{A}_0^{-1}((\mathbf{M}_{1,1} \otimes \mathbf{S}_{1,1})\mathbf{A}_1)$ uniform

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secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$

$\mathbf{A}_0^{-1}$(uniform) uniform

$\mathbf{A}_0^{-1}$(uniform) uniform

lemma. $\approx_c$ random, for **permutation** matrices

proof. $\longleftarrow$ **[BVWW16]**

secure $\mathcal{O}$(permutation)

[Canetti Chen 17, GKW17, WZ17]

$\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$

uniform uniform

uniform uniform

lemma. $\approx_c$ random, for **permutation** matrices

proof. $\longleftarrow$ **[BVWW16]**

$\mathcal{O}$(all-reject) **revisited**

[Chen Vaikuntanathan W 18]

all-reject with **non-permutation** matrices

input.

$$\mathbf{u} = \begin{pmatrix} \star & 1 \end{pmatrix}, \; \mathbf{M}_{i,b} = \begin{pmatrix} \hat{\mathbf{M}}_{i,b} & \\ & 1 \end{pmatrix}$$

$$\forall\, \mathbf{x} : \mathbf{u}\mathbf{M}_{\mathbf{x}} = \begin{pmatrix} \star & 1 \end{pmatrix}$$

$\mathcal{O}$(all-reject) **revisited**

[Chen Vaikuntanathan W 18]

all-reject with **non-permutation** matrices

corollaries.

– improved **efficiency** for constrained PRFs, lockable obfuscation, ...

– key-homomorphic private constrained PRFs

$\mathcal{O}$(all-reject) **revisited**

[Chen Vaikuntanathan W 18]

all-reject with **non-permutation** matrices

- first step towards understanding general matrices
- requires new techniques

$(\mathbf{M} \otimes \mathbf{S})\mathbf{A}$ **not** pseudorandom

new **computational** lemma

$\mathbf{A}^{-1}(\mathbf{Z}+\mathbf{E})$ **hides** $\mathbf{Z}$

$$\boxed{\quad\mathbf{A}\quad}^{-1}\left(\boxed{\quad\mathbf{Z}\quad}+\boxed{\quad\mathbf{E}\quad}\right)$$

new **computational** lemma

$\mathbf{A}^{-1}(\mathbf{Z} + \mathbf{E})$ **hides** $\mathbf{Z}$

$$\boxed{\quad \mathbf{A} \quad}^{-1}\left(\boxed{\quad \mathbf{Z} \quad} + \boxed{\quad \mathbf{E} \quad}\right)$$

idea. embed LWE secret into $\mathbf{A}$

"target switching" in [**Goyal Koppula Waters 18**]

new **computational** lemma

$$\mathbf{A}^{-1}(\mathbf{Z} + \mathbf{E}) \text{ \textbf{hides} } \mathbf{Z}$$

$$\boxed{\mathbf{A}_1 \mid \mathbf{A}_2}^{-1}\left(\boxed{\quad\mathbf{Z}\quad} + \boxed{\quad\mathbf{E}\quad}\right)$$

new **computational** lemma

$$\mathbf{A}^{-1}(\mathbf{Z} + \mathbf{E}) \textbf{ hides } \mathbf{Z}$$

$$\boxed{\mathbf{A}_1 \mid \mathbf{A}_2}^{-1}\left(\boxed{\quad\mathbf{Z}\quad} + \boxed{\quad\mathbf{E}\quad}\right)$$

$$\approx_s \quad \boxed{\begin{matrix} -\mathbf{U} \\ \mathbf{A}_2^{-1}(\mathbf{A}_1\mathbf{U}+\mathbf{Z}+\mathbf{E}) \end{matrix}}$$

new **computational** lemma

$\mathbf{A}^{-1}(\mathbf{Z}+\mathbf{E})$ **hides** $\mathbf{Z}$

$$\boxed{\mathbf{A}_1 \mid \mathbf{A}_2}^{-1}\left(\boxed{\quad\mathbf{Z}\quad} + \boxed{\quad\mathbf{E}\quad}\right)$$

$$\approx_c \quad \boxed{\begin{array}{c} -\mathbf{U} \\ \mathbf{A}_2^{-1}(\text{uniform}) \end{array}}$$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0$$

$$\underbrace{\mathbf{A}_0^{-1}\left(\begin{pmatrix}\hat{\mathbf{M}}_{1,0} & \\ & \mathbf{S}_{1,0}\end{pmatrix}\mathbf{A}_1\right)}_{\sim\sim} \quad \underbrace{\mathbf{A}_1^{-1}\left(\begin{pmatrix}\hat{\mathbf{M}}_{2,0} & \\ & \mathbf{S}_{2,0}\end{pmatrix}\mathbf{A}_2\right)}_{\sim\sim}$$

$$\underbrace{\mathbf{A}_0^{-1}\left(\begin{pmatrix}\hat{\mathbf{M}}_{1,1} & \\ & \mathbf{S}_{1,1}\end{pmatrix}\mathbf{A}_1\right)}_{\sim\sim} \quad \underbrace{\mathbf{A}_1^{-1}\left(\begin{pmatrix}\hat{\mathbf{M}}_{2,1} & \\ & \mathbf{S}_{2,1}\end{pmatrix}\mathbf{A}_2\right)}_{\sim\sim}$$

lemma. $\approx_c$ random, for **any** matrices $\hat{\mathbf{M}}_{i,b}$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0,\ \ \mathbf{S}_{1,b},\ \ \mathbf{S}_{2,b},\ \ \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\Big(\underbrace{\begin{pmatrix} \hat{\mathbf{M}}_{1,0} & \\ & \mathbf{S}_{1,0} \end{pmatrix}\mathbf{A}_1}\Big) \quad \mathbf{A}_1^{-1}\Big(\underbrace{\begin{pmatrix} \hat{\mathbf{M}}_{2,0} & \\ & \mathbf{S}_{2,0} \end{pmatrix}\mathbf{A}_2}\Big)$$

$$\mathbf{A}_0^{-1}\Big(\underbrace{\begin{pmatrix} \hat{\mathbf{M}}_{1,1} & \\ & \mathbf{S}_{1,1} \end{pmatrix}\mathbf{A}_1}\Big) \quad \mathbf{A}_1^{-1}\Big(\underbrace{\begin{pmatrix} \hat{\mathbf{M}}_{2,1} & \\ & \mathbf{S}_{2,1} \end{pmatrix}\mathbf{A}_2}\Big)$$

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[Chen Vaikuntanathan W 18]

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$$\mathbf{A}_0^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1 \\ \\ \mathbf{S}_{1,0}\underline{\mathbf{A}}_1 \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2 \\ \\ \mathbf{S}_{2,0}\underline{\mathbf{A}}_2 \end{pmatrix}$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1 \\ \\ \mathbf{S}_{1,1}\underline{\mathbf{A}}_1 \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2 \\ \\ \mathbf{S}_{2,1}\underline{\mathbf{A}}_2 \end{pmatrix}$$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\, \mathbf{A}_0, \;\; \mathbf{S}_{1,b}, \;\; \mathbf{S}_{2,b}, \;\; \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1 \\ \mathbf{S}_{1,0}\underline{\mathbf{A}}_1 \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2 \\ \mathbf{S}_{2,0}\underline{\mathbf{A}}_2 \end{pmatrix}$$

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proof. **(1)** $\longleftarrow$ **(2)** mask $\overline{\mathbf{A}}_0$ **(3)** $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0,\ \ \mathbf{S}_{1,b},\ \ \mathbf{S}_{2,b},\ \ \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \underset{\sim}{\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1} \\ \underset{\sim}{\mathbf{S}_{1,0}\underline{\mathbf{A}}_1} \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \underset{\sim}{\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2} \\ \underset{\sim}{\mathbf{S}_{2,0}\underline{\mathbf{A}}_2} \end{pmatrix}$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \underset{\sim}{\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1} \\ \underset{\sim}{\mathbf{S}_{1,1}\underline{\mathbf{A}}_1} \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \underset{\sim}{\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2} \\ \underset{\sim}{\mathbf{S}_{2,1}\underline{\mathbf{A}}_2} \end{pmatrix}$$

proof. **(1)** $\longleftarrow$ (2) mask $\overline{\mathbf{A}}_0$ $\overline{(3)}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0,\ \ \mathbf{S}_{1,b},\ \ \mathbf{S}_{2,b},\ \ \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \underset{\sim\sim\sim}{\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1} \\ \\ \underset{\sim\sim\sim}{\mathbf{S}_{1,0}\underline{\mathbf{A}}_1} \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \underset{\sim\sim\sim}{\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2} \\ \\ \text{uniform} \end{pmatrix}$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \underset{\sim\sim\sim}{\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1} \\ \\ \underset{\sim\sim\sim}{\mathbf{S}_{1,1}\underline{\mathbf{A}}_1} \end{pmatrix} \qquad \mathbf{A}_1^{-1}\begin{pmatrix} \underset{\sim\sim\sim}{\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2} \\ \\ \text{uniform} \end{pmatrix}$$

proof. **(1)** $\longleftarrow$ (2) mask $\overline{\mathbf{A}}_0$ $\overline{(3)}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0,\;\; \mathbf{S}_{1,b},\;\; \mathbf{S}_{2,b},\;\; \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \underset{\sim}{\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1} \\ \underset{\sim}{\mathbf{S}_{1,0}\underline{\mathbf{A}}_1} \end{pmatrix} \qquad \overline{\mathbf{A}}_1^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2})$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \underset{\sim}{\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1} \\ \underset{\sim}{\mathbf{S}_{1,1}\underline{\mathbf{A}}_1} \end{pmatrix} \qquad \overline{\mathbf{A}}_1^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2})$$

proof. **(1)** $\longleftarrow$ (2) mask $\overline{\mathbf{A}}_0$ $\overline{(3)}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0,\quad \mathbf{S}_{1,b},\quad \mathbf{S}_{2,b},\quad \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1 \\ \mathbf{S}_{1,0}\underline{\mathbf{A}}_1 \end{pmatrix} \qquad \overline{\mathbf{A}}_1^{-1}(\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2)$$

$$\mathbf{A}_0^{-1}\begin{pmatrix} \hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1 \\ \mathbf{S}_{1,1}\underline{\mathbf{A}}_1 \end{pmatrix} \qquad \overline{\mathbf{A}}_1^{-1}(\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2)$$

proof. **(1)** $\longleftarrow$ (2) mask $\overline{\mathbf{A}}_0$ $\overline{(3)}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0, \;\; \mathbf{S}_{1,b}, \;\; \mathbf{S}_{2,b}, \;\; \overline{\mathbf{A}}_2$$

$$\mathbf{A}_0^{-1}\Big(\underbrace{\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1}_{\text{uniform}}\Big) \qquad \overline{\mathbf{A}}_1^{-1}(\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2)$$

$$\mathbf{A}_0^{-1}\Big(\underbrace{\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1}_{\text{uniform}}\Big) \qquad \overline{\mathbf{A}}_1^{-1}(\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2)$$

proof. **(1)** $\longleftarrow$ (2) mask $\overline{\mathbf{A}}_0$ $\;$ $\overline{(3)}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$[\star \mid \mathbf{I}]\,\mathbf{A}_0,\;\; \mathbf{S}_{1,b},\;\; \mathbf{S}_{2,b},\;\; \overline{\mathbf{A}}_2$$

$$\overline{\mathbf{A}}_0^{-1}(\underbrace{\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1}) \qquad \overline{\mathbf{A}}_1^{-1}(\underbrace{\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2})$$

$$\overline{\mathbf{A}}_0^{-1}(\underbrace{\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1}) \qquad \overline{\mathbf{A}}_1^{-1}(\underbrace{\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2})$$

proof. **(1)** $\longleftarrow$ (2) mask $\overline{\mathrm{A}}_0$ $\overline{(3)}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$\underline{\mathbf{A}}_0, \ \mathbf{S}_{1,b}, \ \mathbf{S}_{2,b}, \ \overline{\mathbf{A}}_2$$

$$\overline{\mathbf{A}}_0^{-1}(\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1) \qquad \overline{\mathbf{A}}_1^{-1}(\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2)$$

$$\overline{\mathbf{A}}_0^{-1}(\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1) \qquad \overline{\mathbf{A}}_1^{-1}(\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2)$$

proof. **(1)** $\longleftarrow$ **(2)** mask $\overline{\mathbf{A}}_0$ (3) $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$\underline{\mathbf{A}}_0,\ \mathbf{S}_{1,b},\ \mathbf{S}_{2,b},\ \overline{\mathbf{A}}_2$$

$$\overline{\mathbf{A}}_0^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{1,0}\overline{\mathbf{A}}_1}) \qquad \overline{\mathbf{A}}_1^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2})$$

$$\overline{\mathbf{A}}_0^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{1,1}\overline{\mathbf{A}}_1}) \qquad \overline{\mathbf{A}}_1^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2})$$

proof. **(1)** $\longleftarrow$ **(2)** mask $\overline{\mathbf{A}}_0$ **(3)** $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$\underline{\mathbf{A}}_0,\ \mathbf{S}_{1,b},\ \mathbf{S}_{2,b},\ \overline{\mathbf{A}}_2$$

uniform $\qquad \overline{\mathbf{A}}_1^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{2,0}\overline{\mathbf{A}}_2})$

uniform $\qquad \overline{\mathbf{A}}_1^{-1}(\underset{\sim}{\hat{\mathbf{M}}_{2,1}\overline{\mathbf{A}}_2})$

proof. **<u>(1)</u>** $\longleftarrow$ **(2)** mask $\overline{\mathbf{A}}_0$ $\overline{\textbf{(3)}}$ $\longrightarrow$

secure $\mathcal{O}$(non-permutation)

[Chen Vaikuntanathan W 18]

$$\underline{\mathbf{A}}_0,\ \mathbf{S}_{1,b},\ \mathbf{S}_{2,b},\ \overline{\mathbf{A}}_2$$

uniform uniform

uniform uniform

proof. **(1)** $\longleftarrow$ **(2)** mask $\overline{\mathbf{A}}_0$ **(3)** $\longrightarrow$

❷ attacks

$\mathcal{O}$(read-**once**)

[Halevi Halevi Stephens-Davidowitz Shoup 17, ...]

input. read-**once** program $\mathbf{u}$, $\{\mathbf{M}_{i,b}\}$

output.

$$(\mathbf{u} \otimes \mathbf{I})\mathbf{A}_0, \ \{ \mathbf{A}_{i-1}^{-1}(\underset{\sim}{(\mathbf{M}_{i,b} \otimes \mathbf{S}_{i,b})\mathbf{A}_i}) \}_{i \in [\ell], b \in \{0,1\}}$$

evaluation. accept if $\underset{\sim}{(\mathbf{u}\mathbf{M}_{\mathbf{x}} \otimes \mathbf{S}_{\mathbf{x}})\mathbf{A}_\ell} \stackrel{?}{\approx} \mathbf{0}$

rank **attack**

[Chen Vaikuntanathan W 18]

1. $$\textbf{eval}(x_i \mid y_j) \approx 0, \quad i,j \in [L]$$

   L^2 accepting inputs $x_i \mid y_j$

starting point

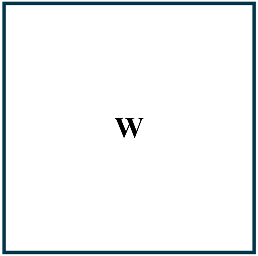
[CHLRS15, CLLT16, CGH17]

rank **attack**

[Chen Vaikuntanathan W 18]

1. $w_{ij} := \textbf{eval}(x_i \mid y_j) \approx 0, \quad i,j \in [L]$

2. $\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L}$



starting point

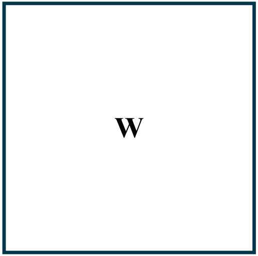
[CHLRS15, CLLT16, CGH17]

rank **attack**

[Chen Vaikuntanathan W 18]

1. $w_{ij} := \textbf{eval}(x_i \mid y_j) \approx 0, \quad i, j \in [L]$

2. $\textbf{rank}(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L})$



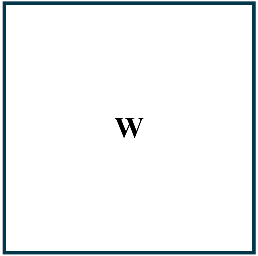
starting point

[CHLRS15, CLLT16, CGH17]

rank **attack**

[Chen Vaikuntanathan W 18]

1. $w_{ij} := \textbf{eval}(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. $\textbf{rank}(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L})$



starting point
[CHLRS15, CLLT16, CGH17]

rank **attack**

[Chen Vaikuntanathan W 18]

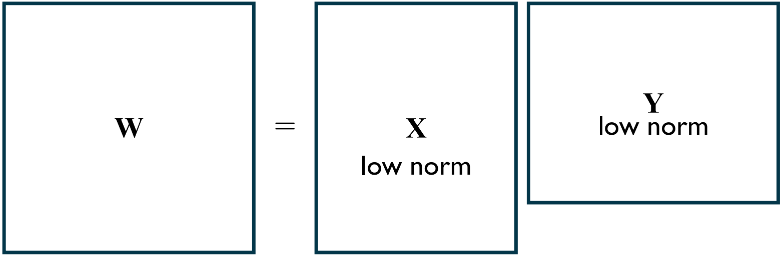
1. $w_{ij} :=$ **eval**$(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. **rank**$(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L})$

$$\boxed{\mathbf{W}} = \boxed{\begin{matrix} - & \hat{\mathbf{x}}_1 & - \\ - & \hat{\mathbf{x}}_2 & - \\ & \vdots & \\ - & \hat{\mathbf{x}}_L & - \end{matrix}} \boxed{\begin{matrix} | & | & & | \\ \hat{\mathbf{y}}_1 & \hat{\mathbf{y}}_2 & \cdots & \hat{\mathbf{y}}_L \\ | & | & & | \end{matrix}}$$

rank **attack**

[Chen Vaikuntanathan W 18]

1. $w_{ij} := \textbf{eval}(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. $\textbf{rank}(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L})$



rank **attack**

[Chen Vaikuntanathan W 18]

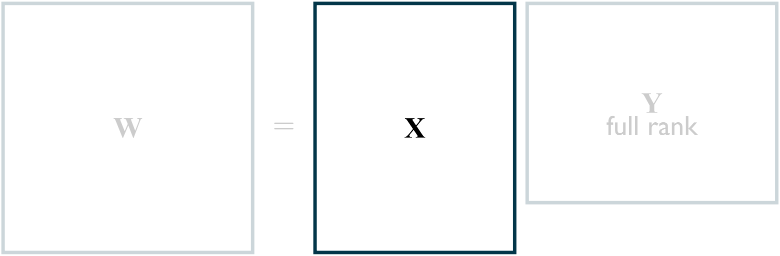
1. $w_{ij} := \textbf{eval}(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. $\textbf{rank}(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L})$

$\mathbf{W}$ $=$ $\mathbf{X}$ low norm $\mathbf{Y}$ full rank

rank **attack**

[Chen Vaikuntanathan W 18]

1. $w_{ij} := \textbf{eval}(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. $\textbf{rank}(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L}) = \textbf{rank}(\mathbf{X})$



rank **attack**

[**Chen Vaikuntanathan W 18**]

1. $w_{ij} := \textsf{eval}(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. $\textsf{rank}(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L}) = \textsf{rank}(\mathbf{X})$

$$\boxed{\mathbf{W}} = \boxed{\begin{matrix} \mathbf{u}\mathbf{M}_{x_1} \otimes \mathbf{S}_{x_1} \mid \mathbf{e}_1 \\ \mathbf{u}\mathbf{M}_{x_2} \otimes \mathbf{S}_{x_2} \mid \mathbf{e}_2 \\ \vdots \\ \mathbf{u}\mathbf{M}_{x_L} \otimes \mathbf{S}_{x_L} \mid \mathbf{e}_L \end{matrix}} \boxed{\begin{matrix} \mathbf{Y} \\ \text{full rank} \end{matrix}}$$

rank **attack**

[Chen Vaikuntanathan W 18]

1. $w_{ij} :=$ **eval**$(x_i \mid y_j) = \langle \hat{\mathbf{x}}_i, \hat{\mathbf{y}}_j \rangle$ assuming read-once
2. **rank**$(\mathbf{W} = (w_{ij}) \in \mathbb{Z}^{L \times L}) =$ **rank**$(\mathbf{X})$

$$\boxed{\mathbf{W}} = \boxed{\begin{matrix} \mathbf{uM}_{x_1} \\ \mathbf{uM}_{x_2} \\ \vdots \\ \mathbf{uM}_{x_L} \end{matrix}} \boxed{\begin{matrix} \mathbf{Y} \\ \text{full rank} \end{matrix}}$$

rank **attack**

[Chen Vaikuntanathan W 18]

read-**many**

- $O(\text{size}^c)$ attack for read-c **[ADGM17, CLTT17]**

rank **attack**

[Chen Vaikuntanathan W 18]

read-**many**

- $O(\text{size}^c)$ attack for read-c **[ADGM17, CLTT17]**
- can be avoided by setting c very large

❸ **candidate**

obfuscating NC^1

simple obfuscation candidate

[Chen Vaikuntanathan W 18]

input. read-**many** program $\mathbf{u}, \{\mathbf{M}_{i,b}\}$

output.

$$(\mathbf{u} \otimes \mathbf{I})\mathbf{A}_0, \ \{ \mathbf{A}_{i-1}^{-1}(\underset{\sim}{(\mathbf{M}_{i,b} \otimes \mathbf{S}_{i,b})\mathbf{A}_i}) \}_{i \in [\ell], b \in \{0,1\}}$$

simple obfuscation candidate

[Chen Vaikuntanathan W 18]

input. read-**many** program $\mathbf{u}$, $\{\mathbf{M}_{i,b}\}$

output.

$$(\hat{\mathbf{u}} \otimes \mathbf{I})\mathbf{A}_0, \ \{ \mathbf{A}_{i-1}^{-1}((\hat{\mathbf{M}}_{i,b} \otimes \mathbf{S}_{i,b})\mathbf{A}_i) \}_{i \in [\ell], b \in \{0,1\}}$$

simple obfuscation candidate

[Chen Vaikuntanathan W 18]

input. read-**many** program $\mathbf{u}$, $\{\mathbf{M}_{i,b}\}$

output.

$$(\hat{\mathbf{u}} \otimes \mathbf{I})\mathbf{A}_0, \; \{ \mathbf{A}_{i-1}^{-1}((\hat{\mathbf{M}}_{i,b} \otimes \mathbf{S}_{i,b})\mathbf{A}_i) \}_{i \in [\ell], b \in \{0,1\}}$$

$$\hat{\mathbf{M}}_{i,b} = \begin{pmatrix} \mathbf{M}_{i,b} & & & \\ & \mathbf{R}_{i,b}^{(1)} & & \\ & & \ddots & \\ & & & \mathbf{R}_{i,b}^{(\ell)} \end{pmatrix}$$

simple obfuscation candidate

[Chen Vaikuntanathan W 18]

input. read-**many** program $\mathbf{u}$, $\{\mathbf{M}_{i,b}\}$

output.

$$(\hat{\mathbf{u}} \otimes \mathbf{I})\mathbf{A}_0, \ \{ \mathbf{A}_{i-1}^{-1}((\hat{\mathbf{M}}_{i,b} \otimes \mathbf{S}_{i,b})\mathbf{A}_i) \}_{i\in[\ell],b\in\{0,1\}}$$

$$\hat{\mathbf{M}}_{i,b} = \begin{pmatrix} \mathbf{M}_{i,b} & & & \\ & \mathbf{R}_{i,b}^{(1)} & & \\ & & \ddots & \\ & & & \mathbf{R}_{i,b}^{(\ell)} \end{pmatrix}$$

$\mathbf{R}_{i,b}^{(j)} \in \{0,1\}^{2\times 2}$
input consistency

simple obfuscation candidate

[Chen Vaikuntanathan W 18]

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status.

– **secure** in idealized model **[Bartusek Guan Ma Zhandry 18]**

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– tweaks against statistical tests **[Cheon Cho Hhan Kim Lee 19]**

❹ obfuscation

some thoughts

obfuscation: small steps

1. **weaker** primitives from LWE

– lockable obfuscation, mixed FE, ...

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// merci !